

I B.Tech Supplementary Examinations, Aug/Sep 2008

MATHEMATICS-I

(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering, Bio-Technology and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Test the convergence of the series

$$x^2 + \frac{2^2 \cdot x^4}{3 \cdot 4} + \frac{2^2 \cdot 4^2 \cdot x^6}{3 \cdot 4 \cdot 5} + \frac{2^2 \cdot 4^2 \cdot 6^2 \cdot x^8}{3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} + \dots (x > 0)$$
 [5]
 (b) Examine whether the following series is absolutely convergent or conditionally convergent $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \dots$ [5]
 (c) Write Taylor's series for $f(x) = (1 - x)^{5/2}$ with Lagrange's form of remainder upto 3 terms in the interval $[0,1]$. [6]
2. (a) If $z = \log(e^x + e^y)$
 show that $rt - s^2 = 0$ where $r = \frac{\partial^2 z}{\partial x^2}$, $t = \frac{\partial^2 z}{\partial y^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$
 (b) Determine the center of curvature to the curve in parametric form $x = 3t^2$, $y = 3t - t^3$. [8+8]
3. Trace the lemniscate of Bernoulli : $r^2 = a^2 \cos 2\theta$. Prove that the volume of revolution about the initial line is $\frac{\pi a^3}{6\sqrt{2}} [3 \log(\sqrt{2} + 1) - \sqrt{2}]$ [16]
4. (a) Obtain the differential equation of the coaxial circles of the system
 $x^2 + y^2 + 2ax + c^2 = 0$ where c is a constant and a is a variable. [3]
 (b) Solve the differential equation:
 $(x^2 - 2xy + 3y^2) dx + (y^2 + 6xy - x^2) dy = 0$. [7]
 (c) Find the orthogonal trajectory of the family of the cardioids
 $r = a(1 + \cos \theta)$ [6]
5. (a) Solve the differential equation: $y'' + 4y' + 20y = 23 \sin t - 15 \cos t$, $y(0) = 0$, $y'(0) = -1$
 (b) Solve the differential equation: $(2x + 5)^2 \frac{d^2 y}{dx^2} + 6(2x + 5) \frac{dy}{dx} + 8y = 4(2x + 5)$ [8+8]
6. (a) Using Laplace transforms solve the differential equation $\frac{d^2 x}{dt^2} - 2 \frac{dx}{dt} + x = e^t$, given that $x(0) = 2$, $x'(0) = -1$ at $t = 0$

- (b) Evaluate by transforming in to polar co-ordinates $\iint \frac{(x^2-y^2)}{(x^2+y^2)^{3/2}} dy dx$ over the region of the circle $x^2 + y^2 = 2ax$ in the first quadrant. [8+8]
7. (a) If $\mathbf{A}$ is irrotational vector, evaluate $\text{div}(\mathbf{A} \times \mathbf{r})$ where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.
(b) If $\mathbf{F} = xy\mathbf{i} - z\mathbf{j} + x^2\mathbf{k}$ and c is the curve $x = t^2, y = 2t, z = t^3$ from $t=0$ to $t=1$.
Evaluate $\int_c \mathbf{F} \cdot d\mathbf{r}$. [8+8]
8. Verify Stoke's theorem for $\mathbf{F} = x\mathbf{i} + z^2\mathbf{j} + y^2\mathbf{k}$ over the plane surface $x+y+z=1$ lying in the first octant. [16]

I B.Tech Supplementary Examinations, Aug/Sep 2008**MATHEMATICS-I**

(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering, Bio-Technology and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Test the convergence of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \dots (x > 0)$ [5]
 (b) Examine whether the following series is absolutely convergent or conditionally convergent $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots (x > 0)$ [5]
 (c) State and prove Generalized mean value theorem. [6]
2. (a) If $u = f(r, s, t)$ where $r = x/y$, $s = y/z$ and $t = z/x$
 show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$
 (b) State and prove the necessary and sufficient conditions for extrema of a function 'f' of two variables. [8+8]
3. Trace the lemniscate of Bernoulli : $r^2 = a^2 \cos 2\theta$. Prove that the volume of revolution about the initial line is $\frac{\pi a^3}{6\sqrt{2}} [3 \log(\sqrt{2} + 1) - \sqrt{2}]$ [16]
4. (a) Form the differential equation by eliminating the arbitrary constant
 $y \cos x = c + e^x$ [3]
 (b) Solve the differential equation:
 $(e^y + y \cos xy) dx + (x e^y + x \cos xy) dy = 0$ [7]
 (c) Obtain the orthogonal trajectories of the family $r(1 + \cos \theta) = 2a$. [6]
5. (a) Solve the differential equation: $(D^2 + 1)y = e^{-x} + x^3 + e^x \sin x$.
 (b) Solve the differential equation: $(D^2 + 1)y = x \sin x$ by variation of parameters method. [8+8]
6. (a) State and prove second shifting theorem. [5]
 (b) Find the inverse Laplace Transformation of $\left[\frac{s+3}{(s^2+6s+13)^2} \right]$ [5]
 (c) Evaluate $\int \int \int z^2 dx dy dz$ taken over the volume bounded by $x^2 + y^2 = a^2$, $x^2 + y^2 = z$ and $z = 0$. [6]

7. (a) Evaluate $\nabla^2 \log r$ where $r = \sqrt{x^2 + y^2 + z^2}$
- (b) Find constants a, b, c so that the vector $\mathbf{A} = (x+2y+az)\mathbf{i} + (bx-3y-z)\mathbf{j} + (4x+cy+2z)\mathbf{k}$ is irrotational. Also find φ such that $\mathbf{A} = \nabla\phi$. [8+8]
8. (a) Apply Green's theorem to evaluate $\oint_C (2xy - x^2)dx + (x^2 + y^2)dy$, where "C" is bounded by $y = x^2$ and $y^2 = x$.
- (b) Apply Stoke's theorem to evaluate $\int_C (y dx + z dy + x dz)$ where 'C' is the curve of the intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. [8+8]

I B.Tech Supplementary Examinations, Aug/Sep 2008

MATHEMATICS-I

(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering, Bio-Technology and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Test the convergence of the series $\sum (n+1)^n \frac{x^n}{n^{n+1}}, (x > 0)$ [5]
- (b) Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{\cos^2 nx}{n\sqrt{n}}$ converges absolutely for all values of x. [6]
- (c) Verify Rolle's theorem for $f(x) = 2x^3 + x^2 - 4x - 2$ in $(-\sqrt{3}, +\sqrt{3})$ [5]
2. (a) Locate the stationary points and examine their nature of the following functions: $u = x^3y^2(12-x-y); (x>0, y>0)$
- (b) Find the envelope of the family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where the two parameters are connected by the relation $a + b = c$ where c is a constant. [8+8]
3. (a) Trace the curve $9ay^2 = (x - 2a)(x - 5a)^2$.
- (b) Find the volume of the solid generated by revolving the lemniscate $r^2 = a^2 \cos 2\theta$ about the line $\theta = \frac{\pi}{2}$. [8+8]
4. (a) Form the differential equation by eliminating the arbitrary constant $y = \frac{a+x}{x^2+1}$. [3]
- (b) Solve the differential equation: $x \frac{dy}{dx} + y = x^3y^6$. [7]
- (c) The temperature of cup of coffee is 92°C , when freshly poured the room temperature being 24°C . In one minute it was cooled to 80°C . how long a period must elapse, before the temperature of the cup becomes 65°C . [6]
5. (a) Solve the differential equation: $(D^2 + 5D + 6)y = e^x$.
- (b) Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2} + 4y = \tan 2x$ [8+8]

6. (a) Find $L [t^2 \sin 2t]$ [5]
 (b) Find $L^{-1} \left[\frac{(s+5)}{s^2-6s+25} \right]$ [5]
 (c) Evaluate $\iint_R (x^2 + y^2) dx dy$ over the triangular region R with vertices (0,0) (1,0) and (0,1). [6]
7. (a) Show that $\nabla \times \left[\frac{\bar{A} \times \bar{r}}{r^3} \right] = -\frac{\bar{A}}{r^3} + \left[3\bar{r} \frac{(\bar{A} \cdot \bar{r})}{r^5} \right]$
 (b) If $\bar{F} = yi + x(1 - 2z)j - xyk$ evaluate $\int_S (\nabla \times \bar{F}) \cdot \bar{N} ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ above the xy plane. [8+8]
8. State Green's theorem and verify Green's theorem for $\oint_C [(xy + y^2)dx + x^2 dy]$, where C is bounded by $y = x$ and $y = x^2$. [16]

I B.Tech Supplementary Examinations, Aug/Sep 2008

MATHEMATICS-I

(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering, Bio-Technology and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Test the convergence of the following series $\sum_{n=1}^{\infty} \frac{n!}{3.5.7.....(2n+1)}$ [5]
 (b) Test the following series for absolute /conditional convergence

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(1+2+3+....+n)}{(n+1)^3}$$
 [5]
 (c) Prove that $\tan^{-1} x = \tan^{-1} \pi/4 + \frac{x - \pi/4}{(1+\pi^2/16)} - \frac{\pi(x-\pi/4)^2}{4(1+\frac{\pi^2}{16})^2} + \dots$ [6]
2. (a) If $u = \log (x^2+y^2) + \tan^{-1}(y/x)$ prove that $u_{xx} + u_{yy} = 0$.
 (b) Define curvature, center of curvature, radius of curvature and circle of curvature. [8+8]
3. (a) Trace the curve : $x = a (\theta + \sin\theta)$; $y = a (1 + \cos\theta)$. Obtain the length of one arch of the curve.
 (b) A sphere of radius 'a' units is divided into two parts by a plane distant (a/2) from the centre. Show that the ratio of the volumes of the two parts is 5 : 27. [8+8]
4. (a) Form the differential equation by eliminating the arbitrary constant 'c':
 $y = 1 + x^2 + c\sqrt{1+x^2}$. [3]
 (b) Solve the differential equation:
 $\frac{dy}{dx} + (y-1) \cos x = e^{-\sin x} \cos^2 x$. [7]
 (c) An object whose temperature is 75°C cools in an atmosphere of constant temperature 25 °C at the rate $k\theta$, θ being the excess temperature of the body over the atmosphere. If after 10 minutes the temperature of the objects falls to 65 °C . Find its temperature after 20 minutes. Find the time required to cool down to 55 °C. [6]
5. (a) Solve the differential equation: $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$

- (b) Solve the differential equation: $(D - 2)y = 8(e^{2x} + \sin 2x + x^2)$ [8+8]
6. (a) Find the Laplace Transformation of the following function
 $e^{-3t}(2\cos 5t - 3\sin 5t)$ [5]
- (b) State and prove convolution theorem to find the inverse of Laplace transforms. [5]
- (c) Use convolution theorem to find
 $L^{-1} \left[\frac{16}{(s^2+4)(s^2+4)} \right]$ [6]
7. (a) Prove that $\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} + (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{B} \times (\nabla \times \mathbf{A}) + \mathbf{A} \times (\nabla \times \mathbf{B})$.
- (b) If $\phi = 2xy^2z + x^2y$, evaluate $\int_C \phi \, d\mathbf{r}$ where C is the curve $x = t, y = t^2, z = t^3$ from $t=0$ to $t=1$. [8+8]
8. Verify Green's theorem for $\oint_C [(3x - 8y^2)dx + (4y - 6xy)dy]$ where C is the region bounded by $x=0, y=0$ and $x + y = 1$. [16]
